

## Steady-State Heat Transfer to Pseudo plastic Fluid Food in Pipe Flow

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### ABSTRACT

The results of numerical simulation of heat transfer to power-law fluids with variable rheology flowing in a cylindrical tube under uniform heat flux at the wall are compared with the results of published experimental study and an established correlation (Filkova *et al.*, 1987), using CMC (Carboxymethylcellulose) solution and Tomato juice as working fluids. Physical properties of tests fluids were assumed to be temperature dependant. The effect of natural convection was checked and considered negligible. The FLUENT 6.1 Package has been used for simulation. The computational model is briefly described. In particular heat transfer coefficients were obtained from the calculated temperature profiles and compared with published flows for CMC solution and tomato juice.

**Key words:** heat transfer, laminar flow, pseudoplastic, numerical modeling.

### INTRODUCTION

The increasing importance of pseudoplastic flow and heat transfer has led to an increase in research activity in this field in recent years. Since such diversified fluids as polymer solutions, polymer melts, paints, foods, rubber solutions, and some biological fluids have been found to exhibit pseudoplastic behavior, consequently, the study of transport phenomena in pseudoplastic fluids is of significant importance. Considerable

study has been undertaken on the subject of heat transfer to low and moderate viscous pseudoplastics fluids in horizontal tubes, both in isothermal and with constant heat flux at wall (Liu J. H. *et al.* 1992).

Mizushina *et al.* (1975) accumulated experimental data on methocel solutions flowing in a circular pipe whose walls were subjected to uniform heat flux. However, Joshi and Bergles (1980) and Scirocco *et al.* (1991) did an experimental investigation related the laminar flow of pseudoplastic solutions in a circular tube subjected to a uniform wall heat flux. The results of Scirocco *et al.* (1991) cover the turbulent regime as well. On the other hand, Lawal and Mujumdar (1985) developed a theoretical model for prediction of heat transfer to variable-property power-law fluids in entrance region of ducts of arbitrary cross-section. Further, studies on heat transfer in carboxymethyl cellulose (CMC) solution and tomato juice flowing in a tube subjected to uniform wall heat flux conditions have been conducted by Koziskova (1983).

The main objectives of this study are to compare the predicted and experimental behavior of carboxymethyl cellulose (CMC) solution and tomato juice, and to determine the temperature distribution along pipe surfaces with a uniform flux heating along the entrance. The data obtained in this study will be used to determine the heat transfer coefficients, Nusselt numbers and to compare the predicted data with established correlation and published data.

## MATERIALS AND METHODS

**Materials:** Two power law-fluids namely (tomato juice and Carboxymethyl cellulose (CMC 1.2%)) were selected for numerical simulation. The thermo-physical properties of the two forementioned fluids are presented in Tables (1 and 2).

Table (1): Temperature and thermo-physical properties of tomato juice and carboxymethyl cellulose solution.

| Test fluid   | Temperature dependent properties   | Units                                     |
|--------------|--|---|
| Tomato juice | $\rho = 1036.97 - 0.45T$<br>$C_p = 3643.9 + 2.63T$<br>$k = 0.570 - 4.44 \times 0.001T$<br>$n = 0.45 \exp(7.1351 \times 0.001T)$<br>$K = 0.728 \exp(-0.05775T)$ | $Kg/m^3$<br>$J/kgK$<br>$W/mK$<br>$Pa s^n$ |
| CMC 1.2%     | $\rho, k$ and $C_p$ same as water Yoo<br>(1975).<br>$n = 0.808 \exp(0.0019495T)$<br>$K = 0.0907 \exp(-0.0408T)$  | $Pa s^n$                                  |

Source: Filkova *et al.* (1987).

$C_p$  (Specific heat capacity (J/kgK);  $k$  (Thermal conductivity (W/mK));  $\rho$  Density (kg/m<sup>3</sup>)  
CMC (Carboxymethylcellulose);  $K$  consistency coefficient;  $n$  flow behavior index;  $T$   
temperature in C. (Gr Pr) = 1867,  $T_w = (40-90) ^\circ C$ ,  $X^* = (0.00001-0.01)$  Filkova *et al.*  
(1987).

$G_z$  (Graetz number ( $mC_p/kx$ ) (dimensionless)) ;  $Pe$  (Peclet number ( $\rho C_p U_e D_h/k$ ) (dimensionless));  $q$  (Heat flux per unit length ( $W/m^2m$ ));  $Re$  (Reynolds number ( $D_h U_e^{2-n} \rho/\mu_0$ ) (dimensionless));

Table (2): Range of experimental conditions for tomato juice and Carboxymethylcellulose solution flow and heat transfer parameters use

|              | Re      | Pe          | Gz     | q ( $W/m^3$ ) |
|--------------|---------|-------------|--------|---------------|
| 1.2% CMC     | 124-988 | 19964-82137 | 83-341 | 8324-9252     |
| Tomato juice | 31-520  | 13158-89428 | 55-372 | 5269-7605     |

### Methods: Model development

The model developed in this study is based on following assumptions:

1. Steady state, laminar and 2-D flow
2. Incompressible fluid with temperature properties dependants
3. No energy dissipation in thermal boundary layer
4. No slip at the cylinder wall
5. No mass flow through cylinder wall
6. Small axial conduction relative to radial conduction
7. Insignificant free convection
8. The center of the pipe is a point of the axisymmetric

### Equations

The steady flow of an incompressible fluid with no external forces is described by the set of three basic equations of continuity, motion, and energy (Bird et al. (2002))

Continuity Equation:

$$(\nabla \cdot \mathbf{v}) = 0 \quad (1)$$

Motion Equation:

$$\rho(\nabla \cdot \mathbf{v})\mathbf{v} = -\nabla P - (\nabla \cdot \boldsymbol{\tau}) \quad (2)$$

Energy Equation:

$$\rho C' (\mathbf{v} \cdot \nabla)\mathbf{v} = -(\nabla \cdot \mathbf{q}') - (\boldsymbol{\tau} : \nabla \mathbf{v}) \quad (3)$$

Where:  $\mathbf{v}$  is the local fluid velocity,  $P$  is the static pressure,  $T$  is the temperature,  $\mathbf{q}'$  is the heat flux vector, and  $\boldsymbol{\tau}$  is the stress tension. The term  $(\boldsymbol{\tau} : \nabla \mathbf{v})$  in the energy equation is the heat production due to viscous dissipation. The role of this term in tubes Newtonian flow has been studied by Brinkman (1951), while its importance in extrusion of molten plastics has been discussed by Bird (1955) under normal conditions. For the liquids flow in cylindrical tubes the above equations may be simplified as follow:

Motion

$$0 = -\frac{\partial P}{\partial z} - \frac{1}{r} \frac{\partial}{\partial r} (r \tau_{rz}) \quad (4)$$

Energy

$$\rho C v_z \frac{\partial T}{\partial z} = -\frac{1}{r} \frac{\partial}{\partial r} (r q_r) \quad (5)$$

Where:  $\tau_{rz}$  is the shear stress in the z-direction per unit area on an element of fluid surface of the constant  $r$ , and  $q_r$  is the heat flux in the radial direction  $r$ . In order to solve these equations we substitute for the fluxes  $\tau_{rz}$  and  $q_r$  the expressions:

Power law:

$$\tau_{rz} = -m \left[ \frac{\partial v_z}{\partial r} \right]^{\frac{1}{n}-1} \frac{\partial v_z}{\partial r} \quad (6)$$

Fourier's law:

$$q_r = -k \frac{\partial T}{\partial r}$$

(7)

Where:  $m$  and  $n$  are constants in the empirical modification of Newton's law, obtainable from analysis of 'flow data in various types of viscosimeters (Reiner (1949)).  $k$  is the coefficient of thermal conductivity.

### The Boundary conditions

The boundary conditions used in this study can be summarized under the following (Fig.,1):

$$T_0 = 313 \text{ } ^\circ\text{K}$$

Heat flux at wall is constant

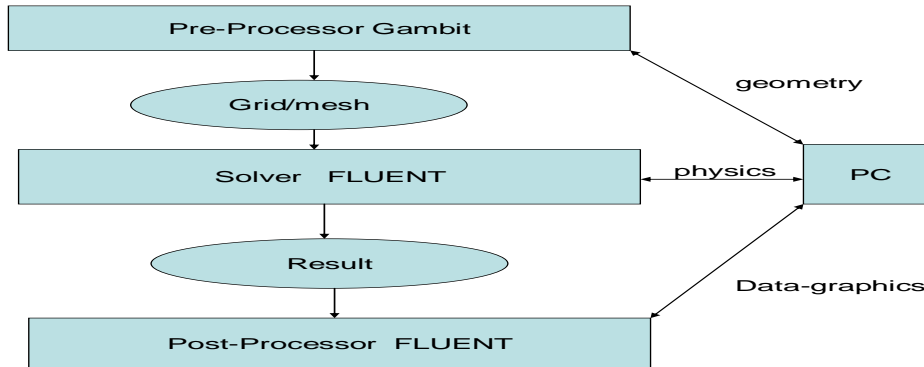


Figure (1). FLUENT flow charge

$$T(r, z, t) = T_0 \text{ for all } r, z \text{ at } t = 0$$

$$V_0 = \text{constant}$$

Pipe is axisymmetric around the center. Figure (1) shows the FLUENT flow charge as expressed by Filkova *et al.* (1987).

The local bulk temperature was determined from the approximate energy balance relating  $T_e$  to the heat input, inlet temperature and flow rate. The wall temperature (after correction for heat conduction across the wall) and the local bulk temperature from the energy balance were combined with the heat input to determine the local Nusselt number.

The widely established correlation for power law fluids Filkova *et al.* (1987). Was selected for prediction of local Nusselt numbers in the thermal entry region of a tube. And it is compared with Predicted data (FLUENT 6.1) and with those obtained experimentally (in detail see Filkova *et al.* (1987).

The correlation equation is:

$$Nu_{x^*} = 1.301 \left( \frac{x}{PeD} \right)^{\frac{1}{3}} \left( \frac{4n}{3n+1} \right)$$

(8)

The axial coordinate is

$$X^* = \left( \frac{x}{PeD} \right) \left( \frac{4n}{3n+1} \right)$$

(9)

Where Pe is Peclet number, D is pipe diameter, x local axial coordinate and n is flow behavior index.

### Numerical study

The one- dimensional Navier –Stoke equations were solved using the commercial software FLUENT. The length of the pipe is 3.02 m and the inside diameter is 0.016 m. A constant velocity with constant temperature was imposed in the entrance boundary, and the constant heat flux was imposed at the wall boundary.

The grid generation software GAMBIT was used for the generating a structured grid in the pipe computational domain. The meshing consists of 25000 cells and the simulations are conducted on half of the domain taking advantage of the symmetry plane.

Steady –state calculations were performed using power law model in FLUENT. The numerical procedure is based on a control volume approach where the computational domain is divided into a number of cells or elements. And the governing equations discretized into algebraic equations within each elements. The control volume approach leads to discretized equations expressing the integral conservation of mass. Momentum and energy over each control volume. The Geometry grid size is  $500 \times 50 = 25000$  cells. It is half of the geometry because of the symmetry about the center.

The quantity of interest for comparison with the measurements of the Nusselt number, Nu, along the heated wall. The Nusselt number was calculated from the bulk temperature and the heat transfer coefficient. The balance between the total heats fluxes transferred from the entrance to the pipe is:

$$T_b(x) = T_e + \left( \frac{\pi \cdot D \cdot q_w \cdot x}{\dot{m} \cdot C_{p_m}} \right) \quad (10)$$

Where  $q_w$  is the local heat flux

The local heat transfer coefficient is:

$$h(x) = \frac{q(x)}{T_{wall}(x) - T_B(x)} \quad (11)$$

Finally, the local Nusselt number is:

$$Nu(x) = \frac{h(x) \cdot D}{k} \quad (12)$$

Where D is the diameter of the pipe and k is the thermal conductivity of the fluids, h(x) local heat transfer coefficient ( $w/m^2 k$ ),  $T_{wall}$  wall temperature (k or  $^{\circ}C$  as appropriate),  $T_b$  Bulk temperature (k or  $^{\circ}C$  as appropriate)

## RESULTS AND DISCUSSION

Figure 2 and 3 show the velocity profile of tomato juice and carboxymethylcellulose respectively. The flat velocity profile decrease towards the outlet of the spipe.

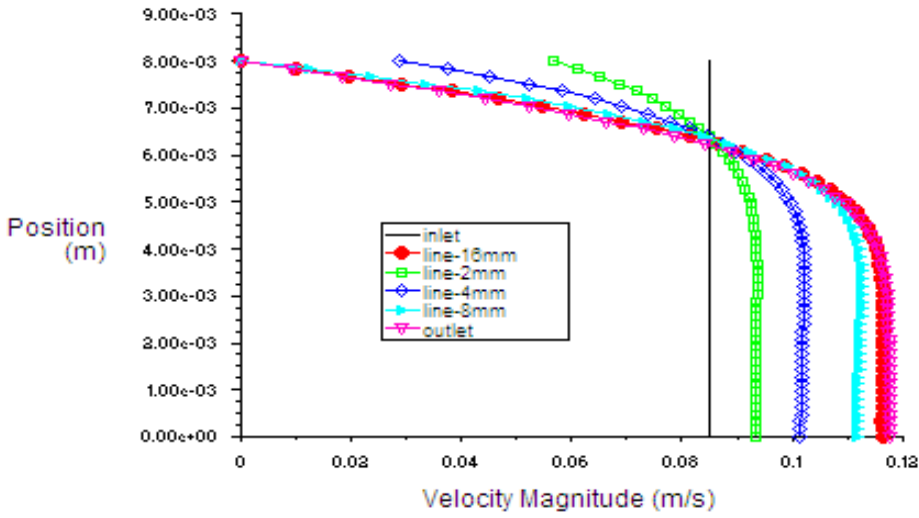


Figure (2) velocity profiles of tomato juice

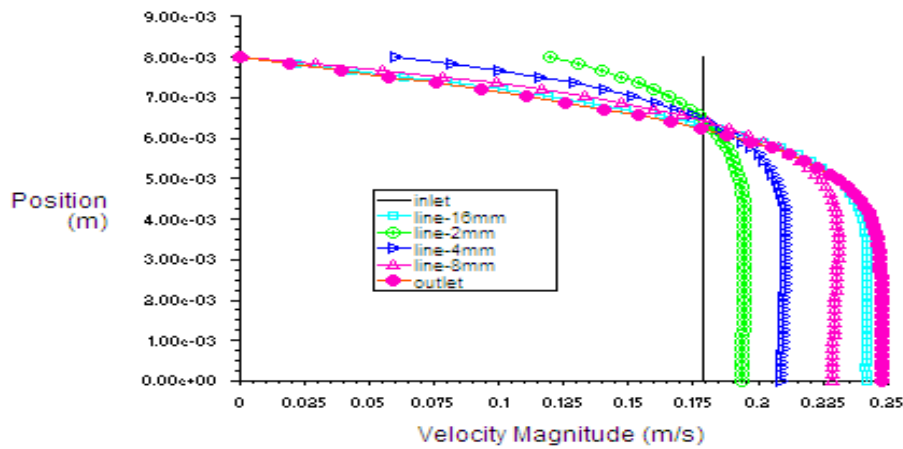


Figure (3) Velocity profiles of 1.2%CMC

Figure 4 and 5 show temperature profiles across the pipe, for different locations from the inlet till the outlet of the pipe for both solutions. However, temperature gradient close to the pipe wall attains higher values for both fluids. The uniformity of the temperature profile across the pipe as the heat diffusion and convection across the pipe enhance equating temperature across the pipe as the residence time increases.

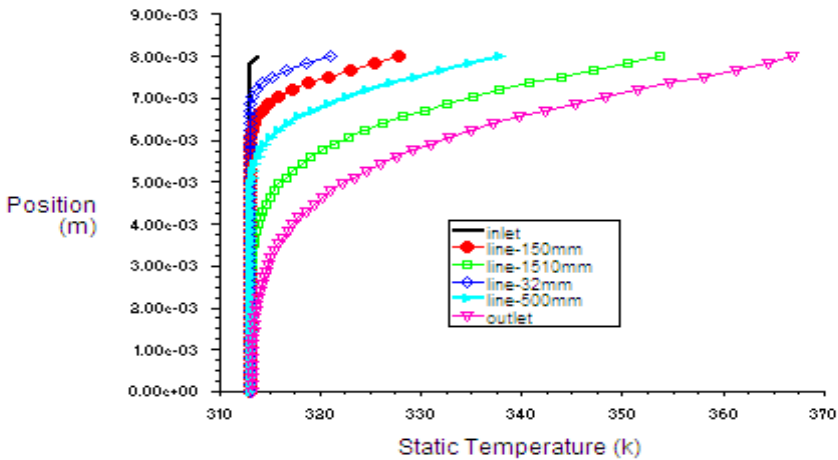


Figure (4) temperature profiles of tomato juice

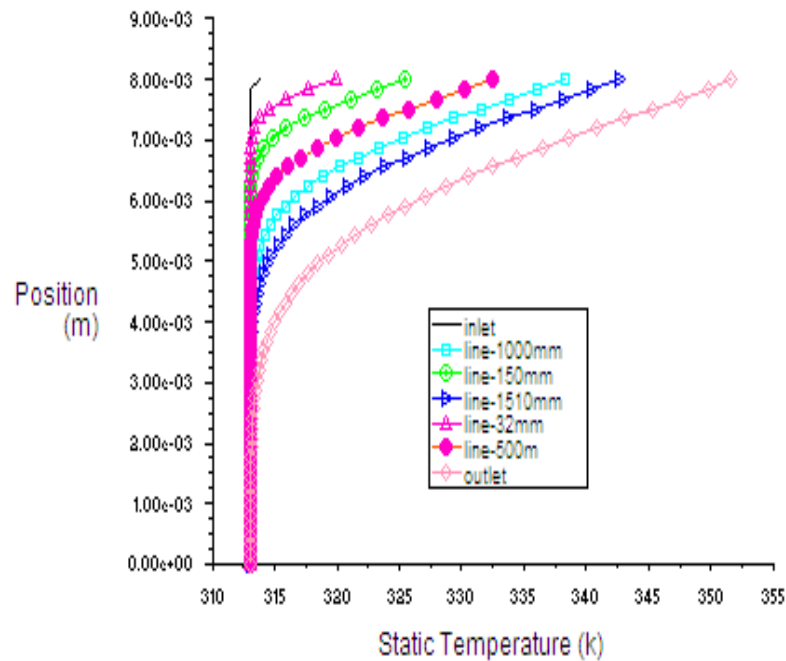
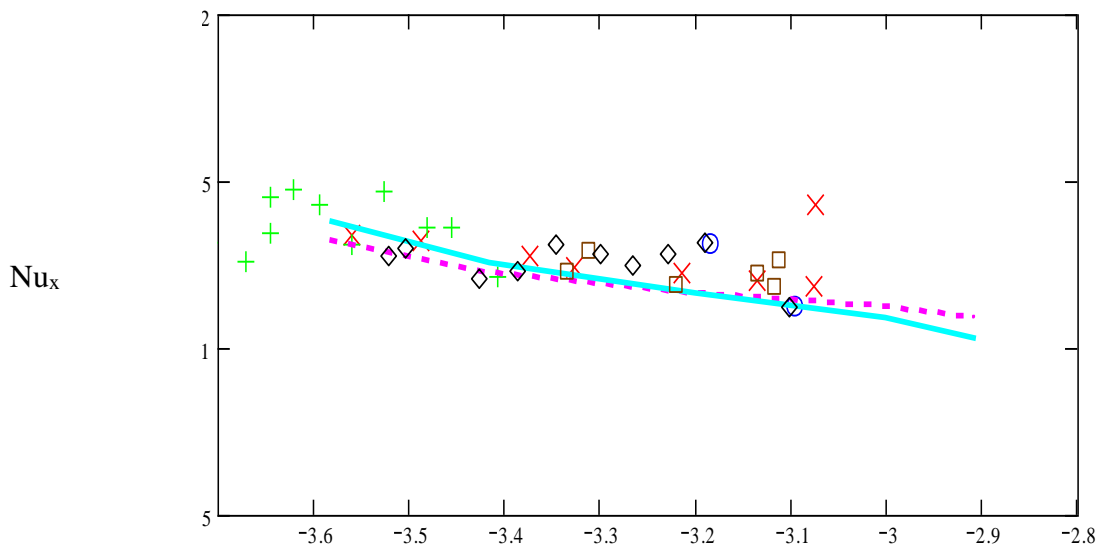


Figure (5) temperature profiles of 1.2% CMC

In Figures 6 and 7 the numerically determined local Nusselt number  $Nu_x$  is plotted against the dimensionless axial coordinate for the two test fluids. Also shown on these figures for the purpose of comparison are experimentally from Filkova *et al.* (1987) paper and from a correlation proposed by Bird (1959). The rheological and thermal properties were evaluated using average bulk temperatures. The range of experimental conditions is given in Table 2. It should be noted that most of the data lie within an axial coordinate  $X^*$  of  $10^{-10}$  and  $10^{-3}$  where the flow is fully developed. In all, experimental run encompassing five different values of  $q$  were made for the two solutions. In general the experimental  $Nu_x$  value is the higher, more so near the entrance to the test section. A possible explanation for the scatter is the difficulty of determining accurately the outlet bulk temperature. Even though a mixer was located inside the transparent chamber, the mixing

elements could presumably be insufficient for adequate mixing. Heat transfer data for tomato juice as the test fluid are plotted in Figure (4). The experimental runs at five different values of  $q$  were made Filkova et al. (1987). It is clearly evident that the scatter in the results is significantly less than that for the CMC solution, an outcome that may be attributable principally to better fluid mixing. The better agreement between experimental and numerical predictions for tomato juice may also be due in part to the fact that the thermo physical properties of tomato juice were experimentally determined, the poorest agreement occurring near the entrance. The value so obtained was generally in good agreement with that calculated from the area integration of the cross-section temperature measurements.



$X^*$

Figure (6) Comparison of numerical heat transfer results on Tomato juice, with correlation equation and published experimental data at different location of the thermocouples: +, X, O,  $\square$ ,  $\diamond$ , Correlation - - - and FLUENT results

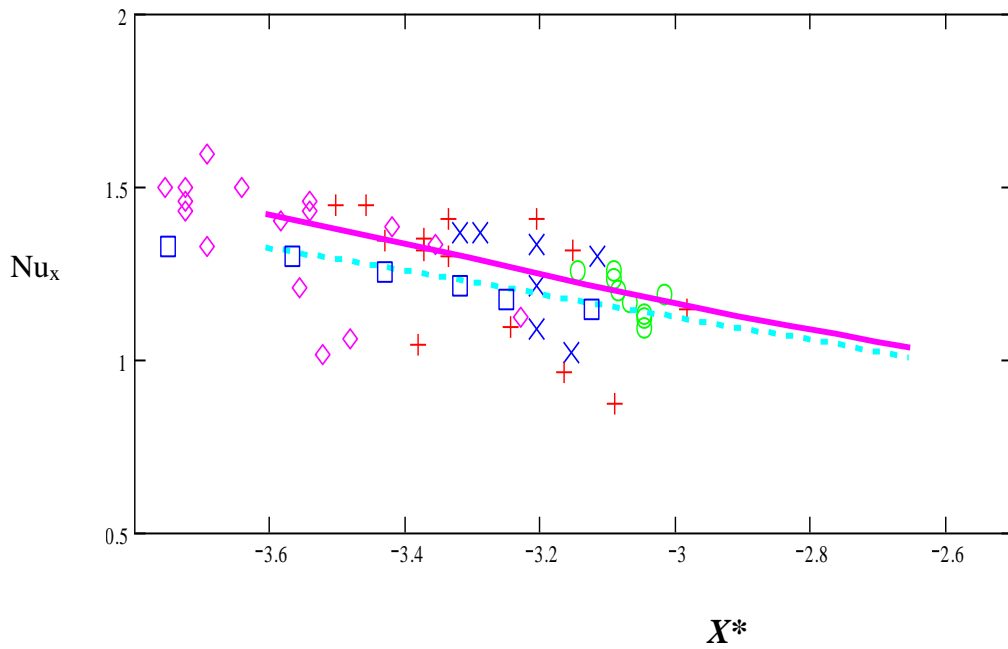


Figure (7) Comparison of numerical heat transfer results on 1.2% CMC with Correlation equation and published experimental data at different location. of the thermocouples: +, X, O,  $\square$ ,  $\diamond$ , Correlation - - - - and FLUENT results

## CONCLUSION

The numerical modeling (CFD-FLUENT) have capability of predicting, with accuracy, heat transfer rates to real, non-Newtonian fluids such as tomato juice flowing in circular pipes.

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### ملخص الدراسة

نتائج المحاكاة الرقمية لإنتقال الحرارة لسوائل تخضع للقانون الأسي وخواصها الريولوجية متغيرة بتغير درجة الحرارة, تجرى داخل انبوية اسطوانية تحت انتقال الحرارة المنتظم عند السطح, تمت مقارنتها مع نتائج الإختبارات المعملية وال Correlations التي تم نشرها بواسطة Filkova وآخرون عام 1987م. تم استخدام محلول CMC وعصير الطماطم كسوائل شغالة. افترض أن الخواص الريولوجية لهذه السوائل تتأثر بالتغير في درجة الحرارة. تم اختبار أثر انتقال الحرارة بالحمل الطبيعي وتم اهماله, تم استخدام برنامج FLUENT-6.1 لعمل المحاكاة. النموذج بإختصار بصورة خاصة يصف معامل انتقال الحرارة المتحصل عليه من خريطة توزيعات الحرارة و تم مقارنتها مع النتائج المنشورة لمحلول CMC وعصير الطماطم.