Using Nyquist Stability Criterion for Determining the Stability of Open and Closed-Loop Control Systems

Abdelrahim B. Hamid, Associate Professor, Department of Mathematics and Physics, Faculty of Education, Hantoub, University of Gezira, Sudan.

ABSTRACT

The Nyquist stability criterion has been applied in polar plots of the open-loop transfer function. In analyzing multiple-loop systems, the inverse transfer functions sometimes may be used in order to permit graphical analysis, this avoids much of numerical calculations. Nyquist stability criterion used to determine the stability of closed-loop system from its open-loop frequency response and open-loop poles.

In this paper we represented some illustrative examples of the stability analysis for control systems using Nyquist stability criterion. In examining the stability of open and closed-loop control systems, we found that three possibilities can occur.

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INTRODUCTION

This section presents mathematical background for understanding the Nyquist stability criterion. Consider the closed-loop system shown in Figures (1), (2). The closed-loop transfer function is:

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}.$$

For stability, all roots of the characteristic equation: 1 + *G(s)H(s)* = 0, must lie in the left-half s plane. It is noted that, although poles and zeros of the open-loop transfer function *G(s)H(s)* may be in the right-half s plane, the system is stable if all the poles of the closed-loop transfer function are in the left-half s plane [3].

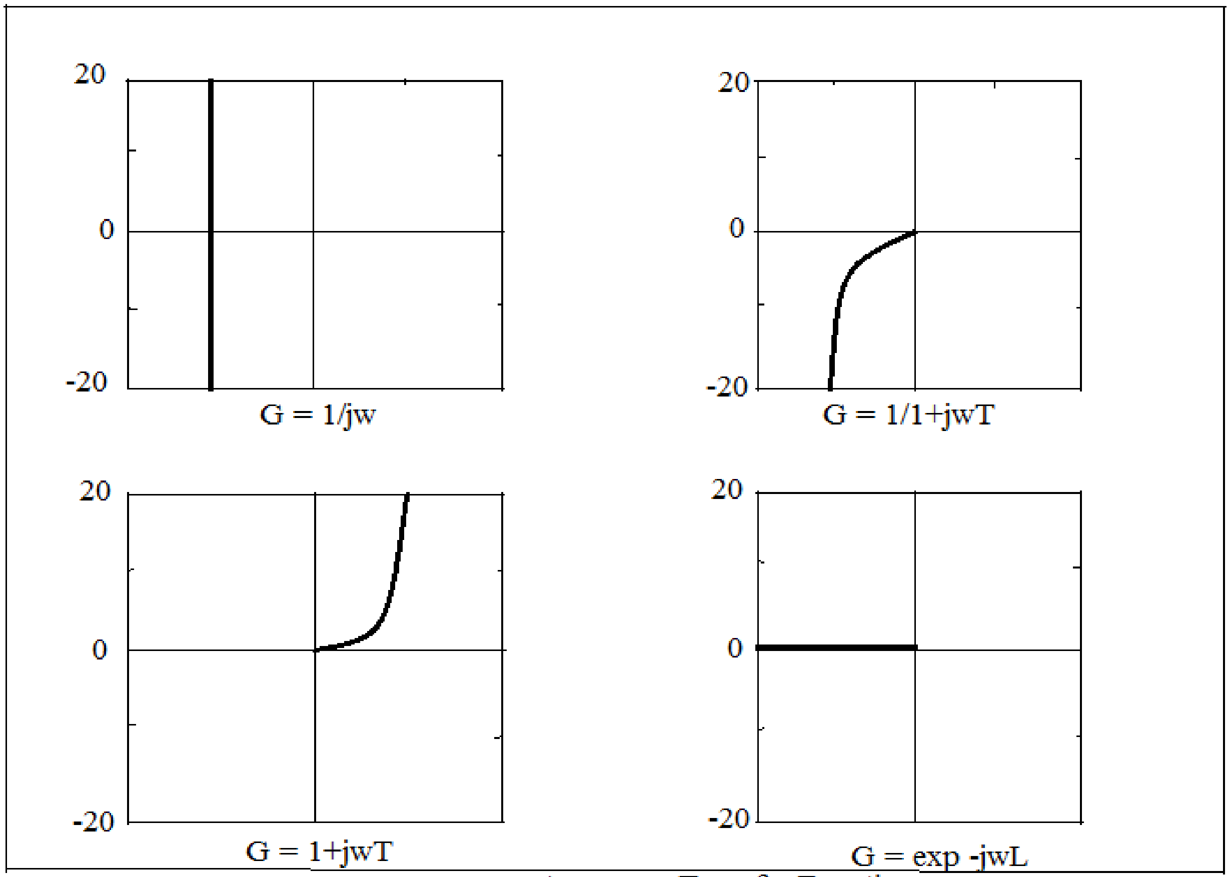


Figure (1) Plots of Transfer Function

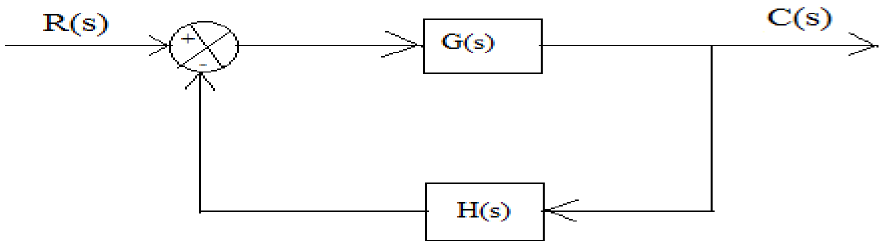


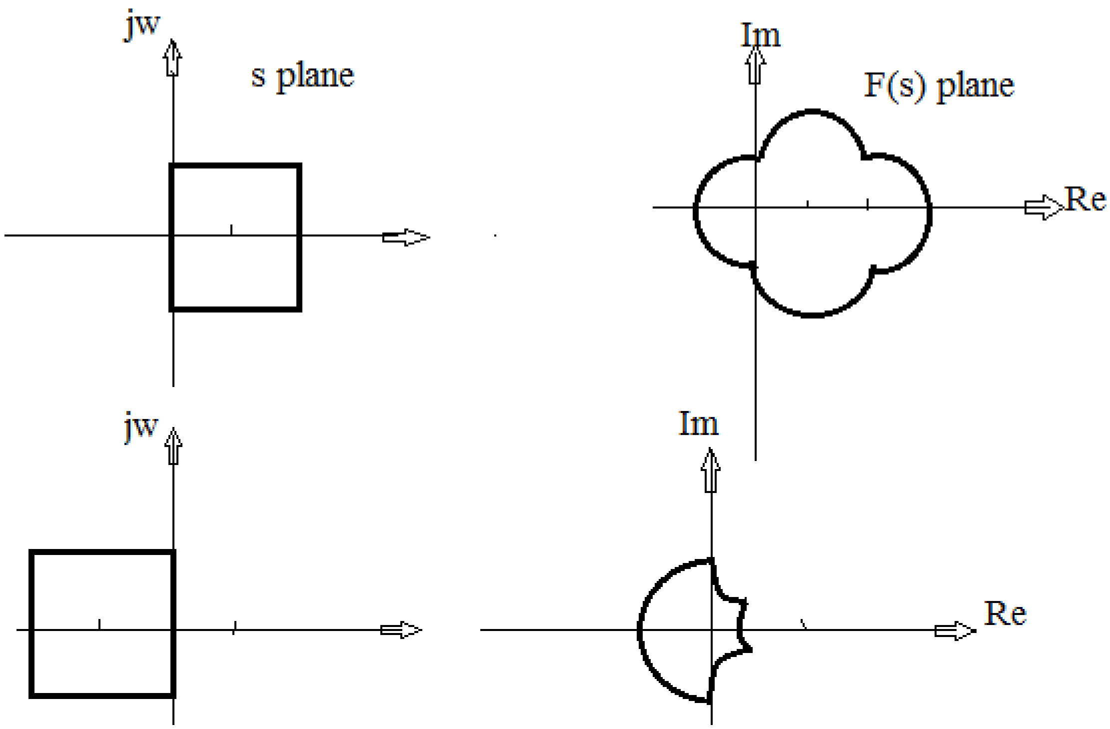
Figure (2) Closed-loop System

The Nyquist stability criterion relates the open-loop frequency response *G(jw)H(jw)* to the number of zeros and poles of 1+*G(s)H(s)* that lie in the right-half s plane. This criterion, derived by H. Nyquist, is useful in control engineering because the absolute stability of the closed-loop system can be determined graphically from open-loop frequency response curves, and there is no need for actually determining the closed-loop poles. Analytically obtained open-loop frequency response curves, as well as those experimentally obtained, can be used for the stability analysis. This is convenient because, in designing control system, it often happens that mathematical expressions for some of the components are not known; only their frequency response data are available [3], [5].

The Nyquist stability criterion is based on a theorem from the theory of complex variables. To understand the criterion, we shall first discus mappings of contours in the complex plane.

We shall assume that the open-loop transfer function *G(s)H(s)* is representable as a ratio of polynomials of s. For physically realizable system, the degree of the denominator polynomial of the closed-loop transfer function must be greater than equal to that of the numerator polynomial. This means that the limit of *G(s)H(s)*
as s approaches infinity is zero or a constant for any physically realizable system.

Suppose that representative point s traces out a contour in the s plane in the clock wise direction if the contour in the s plane encloses the pole of *F(s)*, there is no encirclement of the origin of *F(s)* plane by the locus of *F(s)* in the counter clock wise as appear in Figure (3).



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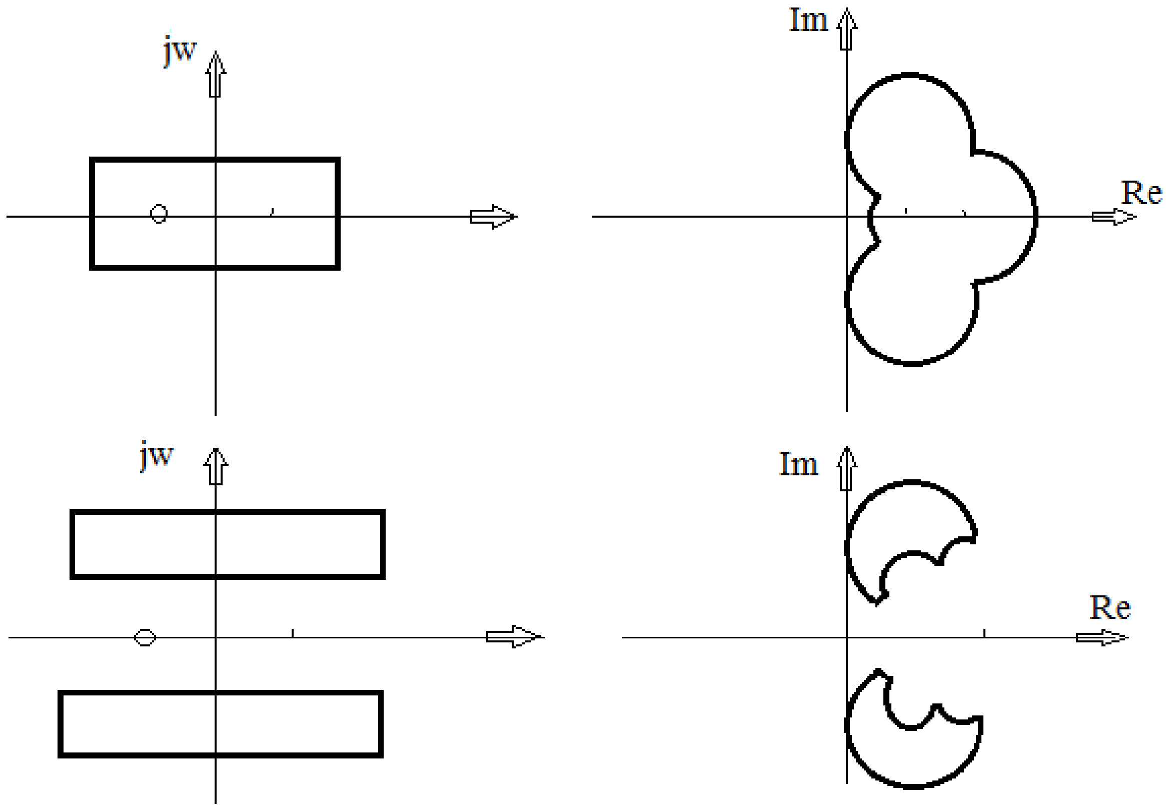

Figure (3) Closed contoursin s plane and their corresponding in F(s) plane

SESULTS AND DISCUSSIONS

Nyquist stability criterion that the foreging analysis, utililizing the encirclement of the -1+j0 point by the *G(jw)H(jw)* locus is defined as; [for special case when *G(s)H(s)* has neither poles nor zeros on the jw-axis, if the open-loop transfer function has k poles in the right-half s plane, and $\lim_{s\to\infty}$ *G(s)H(s)* = constant, then for stability, the *G(jw)H(jw)* locus, as w varies from -∞ to ∞, must encircle the −1+j0 point k times in the counter clock wise direction].

Example 1: Consider a closed-loop system whose open-loop transfer function is given by:

$$G(s)H(s) = \frac{k}{(T_1 s + 1)(T_2 s + 1)}.$$

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Examine the stability of the system. A plot of *G(jw)H(jw)* is shown in Figure (4). Since *G(s)H(s)* does have any poles in the right-half s plane and the −1+j0 point is not encircle by the *G(jw)H(jw)* locus, this system is stable for any positive values of k, T_1, and T_2.

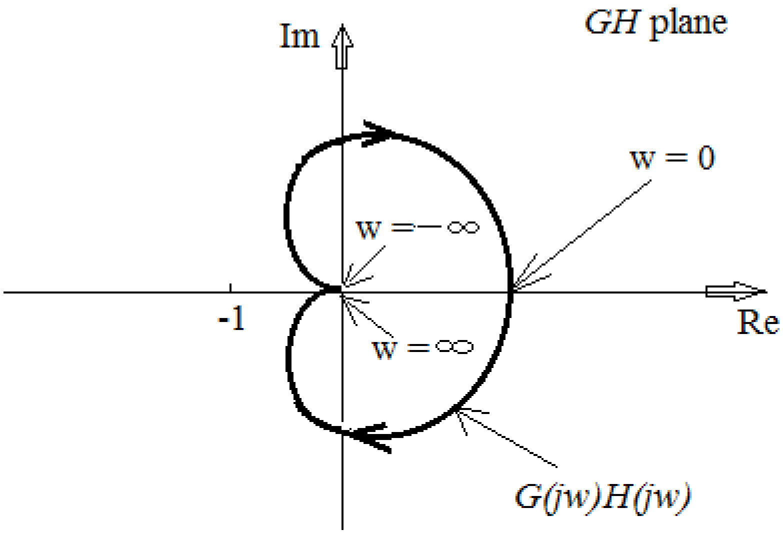

Figure (4) Polar plot of $G(jw)H(jw)$ in Example(1).

Example 2: Consider the closed-loop system having the following open-loop transfer function:

$$G(s)H(s) = \frac{k}{s(Ts-1)}.$$

Determine the stability of the system. The function *G(s)H(s)* has one pole (s = 1/T) in the right-half s plane. Therefore, P = 1. Nyquist plot shown in Figure (5) indicates that the *G(s)H(s)* plot encircles the –1+j0 point once clock wise. Thus, N = 1. Since Z = N + P, we find that Z = 2. This means that the closed-loop system has two closed-loop poles in the right-half s plane and is unstable [1], [4].

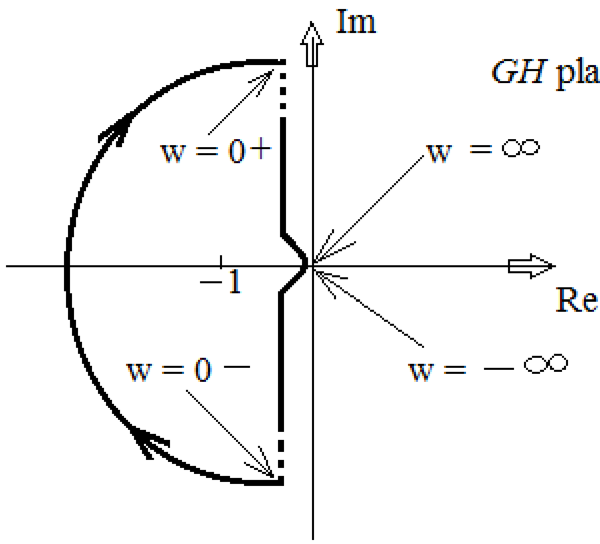

Figure (5) Polar plot cocsidered in Example (2).

Example 3: Investigate the stability of a closed-loop system with the following open-loop transfer function:

$$G(s)H(s) = \frac{k(s+3)}{s(s-1)} \ , \ k > 1.$$

The open-loop transfer function has one pole (p = 1) in the right-half s plane, or P = 1. The open-loop system is unstable. The Nyquist plot shown in Figure (6) indicates that the −1+j0 point is encircled by the *G(s)H(s)* locus once in the counter clock wise direction [6].
Therefore, N = −1. Thus, Z is found from Z = N + P to be zero, which indicates that there no zero of 1 + *G(s)H(s)* in the right-half s plane, and the closed-loop system is stable. This is one of the examples for which an unstable open-loop system becomes stable when the loop is closed [2].

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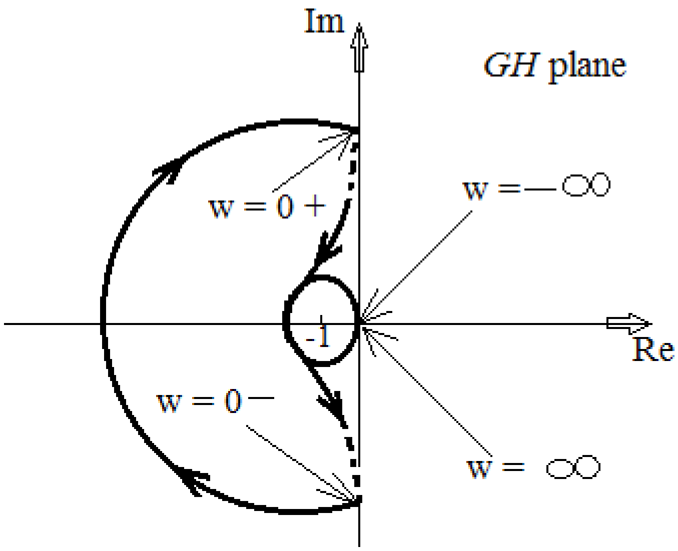

Figure (6) Polar plot of the system in Example (3).

CONCLUSIONS

In examining the stability of linear control systems using Nyquist stability criterion, we found that three possibilities can occur:

(i) There is no encirclement of the −1+j0 point. This implies that the system is stable if there no poles of *G(s)H(s)* in the right-half s plane; otherwise, the system is unstable.

(ii) There are one or more counter clock wise encirclements of the −1+j0 point. In this case the system is stable if the number of counter clock wise encirclements is the same as the number of the poles of *G(s)H(s)* in the right-half s plane; otherwise, the system is an stable.

(iii) There are one or more clock wise encirclements of the −1+j0 point. In this case the system is unstable [6].

We must be careful when testing the stability multiple-loop systems since they may include poles in the right-half s plane. Simple inspection of encirclements of the −1+j0 point by the *G(jw)H(jw)* locus is not sufficient to detect instability in multiple-loop systems.

If the locus of the *G(jw)H(jw)* passes through the −1+j0 point, then zeros of the characteristic equation, or closed-loop poles, are located on the jw-axis. This is not desirable for practical control systems.

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مجلة الجزيرة للهندسة والعلوم التطبيقية المجلد (11) – العدد (1) – 2016م 1437هـ

استخدام مقياس استقرار نايكوست لتحديد استقرار أنظمة التحكم المغلقة والمفتوحة

عبد الرحيم ب. حامد
، أستاذ مشارك ، قسم الرياضيات والفيزياء ، كلية التربية ، حنتوب ، جامعة الجزيرة ، السودان.

مــــلــــخـــــص

يستخدم مقياس استقرار نايكوست في تخطيط المنحنيات القطبية للدوال الناقلة ذات الحلقات المفتوحة. وفي تحليل النظم متعددة الحلقات تستخدم الدوال الناقلة العكسية لرسم منحنيات التحليل، مما يفضي الى حسابات كبيرة ومعقدة. في هذه الورقة تم استخدام مقياس نايكوست للتفريق بين النظام المغلق وتكرار الاستجابة للنظام المفتوح.

ثم عرضت الأمثلة الموضحة لتحليل الاستقرار باستخدام مقياس استقرار نايكويت. وفي اختبار الإستقرارية للأنظمة المغلقة والمفتوحة، ظهرت ثلاث حالات ممكنة الحدوث تم تفصيل كل منها.